M.Sc 4th Semester examination, 2021

Department of Mathematics, Mugberia Gangadhar Mahavidyalaya

(Magneto Hydro Dynamic& Stochastic Process)

Paper: MTM – 403

FULL MARKS : 40

Time : 2 hour

GROUP- A (Magneto Hydro Dynamic)
[Marks-20]

Group- A

Answers any two questions of the following

1. Answer any five questions of the following: 2×5
   a. Define MHD power generator.
   b. What is finch effect.
   c. State Alfven's theorem.
   d. State Maxwell's equations for electromagnetism.
   e. Define magnetic Reynolds number.
   f. State Ferraro's law of isorotation.
   g. Write Navier stokes equation of motion.
   h. Derive differential form of Faraday's law.

2. (a) Solve the problem of MHD flow in a circular pipe with no slip and non-conducting boundary conditions in presence of a uniform transverse magnetic field.

(b)Derive energy of the magnetostatic field. 7+3

3. (a) Define Lorentz force. [2+5+3]

(b) State and prove Ferraro's law of isorotation.

(c) Derive the equations of motion of a conducting fluid.

4. (a) Give the mathematical formulation of MHD flow past a porous plate and derive its velocity expression. [5+5]

(b) Define the terms Alfven's velocity and Alfven's waves. Hence, derive the speed of propagation is $\sqrt{c^2 + V_A{}^2}$ for magneto hydrodynamic wave, where symbols have their usual meaning.

GROUP- B (Stochastic process)

[Marks-20]

Group- B

Answers any two questions of the following

1. (a) State and prove the First Entrance theorem.
   (b) State and prove Chapman Kolmogorov equation. 5+5

2. (a)Show that a I$\in$ S of a Markov chain is recurrent if and only if $\sum_{n=0}^{\infty} P_{ii}^{(n)} = \infty$.

(b)State basic postulates of Poisson process.

(c) Define the graph of a Markov chain. 5+3+2

3. Find the differential equation for birth and death process and hence derive the generating function. 7+3

4. (a) Let (x_i, y_i, z_i), (i=1, 2, 3, …, n)be a sample of size **n**drawn a population. Find the regression equation of **z** on **x** and **y**.

(b) Define multiple and partial co-relation coefficients. Find the relation between them. 6+4

M.Sc 4th Semester Examination, 2021

Department of Mathematics, Mugberia Gangadhar Mahavidyalaya

(Operational Research Modeling-II)

Paper MTM – 405(Unit-I)

FULL MARKS: 20

Time: 1 Hour

Answer any two questions 10×2

1. (a) Find the sequence that minimizes the total elapsed time and idle time required for all three machines to complete the following tasks: 5

Tasks	A	B	C	D	E	F	G
Time on I machine	3	8	7	4	9	8	7
Time on II machine	4	3	2	5	1	4	3
Time on III machine	6	7	5	11	5	6	12

2.

(b) Let X_n be a particular event with probability p_n is distributed into m mutually exclusive sub-events $Y_1, Y_2 ... Y_m$ with probabilities . $q_1, q_2 ... q_m$ respectively, such that $p_n = q_1 + q_2 + ... + q_m$ then

$$H(p_1, p_2, ... p_{n-1}, q_1, q_2, .. q_m) = H(p_1, p_2, ... p_{n-1}, p_n) + p_n H(\frac{q_1}{p_n} + \frac{q_2}{p_n} + ... + \frac{q_m}{p_n}) \qquad 5$$

2. (a) In a system, there are n number of components connected in parallel with reliability $R_i(t)$=n, i=1, 2, ... n. Find the reliability of the system.
If $R_1(t)=R_2(t)= \ldots\ldots = R_n(t) = e^{-\lambda t}$.Then find the reliability of the system.

(b) An electrochemical system is characterized by the ordinary differential equations $\frac{dx_1}{dt} = x_2$ and $\frac{dx_2}{dt} + x_2 = u$ where u is the control variable chosen in such a way that the cost function $\frac{1}{2}\int_0^a (x_1^2 + 4u^2)dt$ is minimized. Show that if the boundary conditions satisfied by the state variables are $x_1(0) = a, x_2(0) = b$, where a, b are constants and $x_1 \to 0, x_2 \to 0$ as $t \to \infty$, the optimal choice for u is $u = -\frac{1}{2}x_1(t) + (1-\sqrt{2})x_2(t)$. 3+7

3. (a) A transmitter and receiver have an information consisting of three letters. The joint probabilities for communication are given below.

$P(x_i, y_j)$	y_1	y_2	y_3
x_1	0.25	0.28	0.05
x_2	0.06	0.12	0.02
x_3	0.04	0.08	0.10

Determine the entropies $H(X), H(Y)$ and $H(X/Y)$ for this channel. 6

(b) Establish the following results for two-dimensional discrete probability distribution

(i) $H(X,Y) = H(X) + H(Y)$ if and only if X and Y are independent.

(ii) $H(X,Y) = H(X|Y) + H(Y) = H(Y|X) + H(X)$. 4

4. (a) Describe the Bang Bang control and illustrate it with the help of an example.

(b) Find the stationary path $x = x(t)$ for the functional $J = \int_0^1 [1 + (\frac{d^2x}{dt^2})]\, dt$, where boundary conditions are $x(0) = 0, x(1) = \dot{x}(0) = \dot{x}(1) = 1$. 6+4

M.Sc.4th Semester examination, 2021

Department of Mathematics, Mugberia Gangadhar Mahavidyalaya

(Functional Analysis)

Paper MTM – 401

FULL MARKS: 40 : : Time : 02 hours

(*Candidates are required to give their answers in their own words as far as practicable*)

*Answer any **Four** of the following questions* 4 X 10

1.	(a) Prove that a subspace M of a Banach space X is complete if and only if M is closed in X. (b) If $f(x) = f(y)$ for every $f \in X^*$, show that $x = y$. (c) If in an inner product space, $\langle x, u\rangle = \langle x, v\rangle$ for all x, show that $u = v$.	6+2+2
2.	(a) If $\{e_1, e_2, e_3, \dots, e_n\}$ is a finite orthonormal set in an inner product space X and x is any element of X, then prove that $\sum_{i=1}^{n} \lvert\langle x, e_j\rangle\rvert^2 \leq \lvert\lvert x\rvert\rvert^2$. When equality holds? (b) Prove that inner product is continuous.	6+4
3.	(a) Let X and Y be two normed linear spaces over the same field of scalars and let $T : X \to Y$ be a linear operator that sends a convergent sequence in X to a bounded sequence in Y. Prove that T is a bounded linear operator. (b) Examine if $T : \mathbb{R}^2 \to \mathbb{R}^2$ be defined by $T(x, y) = (x', y')$, where $x' = x\cos\alpha + y\sin\alpha$ and $y' = -x\sin\alpha + y\cos\alpha$ is a bounded linear transformation.	6+4
4.	(a) Show that in a normed space a linear operator is continuous if and only iff it is bounded. (b) Show that the dual space of a normed space is a Banach space.	6+4
5.	(a) State Open Mapping Theorem. Show that $T : \mathbb{R}^2 \to \mathbb{R}$ be defined by $T(x, y) = x$ for $(x, y) \in \mathbb{R}^2$ is an open mapping. Is T^{-1} bounded, if exists? (b) Suppose $X = C$ [0,1], i.e. the set of all functions $f: [0,1] \to \mathbb{C}$ such that f' exists and is continuous. Let $Y = C[0,1]$ and let X and Y be equipped with supremum norm. Define $A: X \to Y$ by $Af = f'$. Show that the graph of A is closed.	5+5

6.	(a) Show that the space l^p is complete; here p is fixed and $1 \leq p < \infty$. (b) Give an example of an incomplete metric space and justify your answer.	6+4
7.	(a) Let the space $l^2(\mathbb{Z})$ be defined as the space of all two- sided square summable sequences and the bilateral shift is the operator W on $l^2(\mathbb{Z})$ defined by $W(\dots, a_{-2}, a_{-1}, \hat{a}_0, a_1, a_2, \dots) = (\dots, a_{-3}, a_{-2}, \hat{a}_{-1}, a_0, a_1, \dots)$. Prove that (i) W is unitary, and (ii) the adjoint W^* of W is given by $W^*(\dots, a_{-2}, a_{-1}, \hat{a}_0, a_1, a_2, \dots) = (\dots, a_{-1}, a_0, \hat{a}_1, a_2, a_3, \dots)$. (b) Show that $< Ae_j, e_i >= (i + j + 1)^{-1}$ for $0 \leq i, j \leq \infty$ defines a bounded operator on $l^2(\mathbb{N} \cup \{0\})$ with $\parallel A \parallel \leq \pi$.	6+4
8.	(a) Discuss strong convergence and weak convergence. (b) State and prove Uniform Boundedness principle.	3+7

M.Sc.4th Semester examination, 2021

Department of Mathematics, Mugberia Gangadhar Mahavidyalaya

(course name)

Paper MTM – 402

FULL MARKS: 40 : : Time : 02 hours

Unit I: **Fuzzy Mathematics with Applications**

Answer any two questions of the following: **2×10**

1.	(a) Discuss the concept of fuzzy sets with proper example. (b) Prove that $[a_1, b_1, c_1, d_1]+[a_2, b_2, c_2, d_2] = [a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2]$, where $[a, b, c, d]$ is a trapizoidal fuzzy number.	3+7
2.	(a) State Bellman and Zadeh's principle. (b) Consider the LPP-Model $Maximize\ z = 2x_1 + x_2$ $Such\ that\ \ x_1 \lessapprox 3$ $x_1 + x_2 \lessapprox 4$ $5x_1 + x_2 \lessapprox 3$ $x_1, x_2 \geq 0.$ The "tolerance intervals" of the constraints are $p_1 = 6, p_2 = 4, p_3 = 2$. Using Werner's method find its solution.	3+7
3.	(a) Prove that the distributive properties of fuzzy set over standard union and intersection. (b) Let $\tilde{A} = (0, 2, 5)$ and $\tilde{B} = (3, 4, 6)$ be two triangular fuzzy numbers. Find $\tilde{A} \cup \tilde{B}$ and $\tilde{A} \cap \tilde{B}$.	5+5
4.	(a) Let $\tilde{A} = \{(-2, 0.45), (-1, 0.50), (0, 0.80), (1, 1), (2, 0.40)\}$ and $f(x) = x^2$. Find $f(\tilde{A})$. (b) Define a fuzzy multi-objective linear programming problem in general form. (c) Write one of the methodologies to find the deterministic form of a 1st order linear fuzzy differential equation considering it as an initial value problem.	2+4+4

Unit II: **Soft Computing**

Answer any two questions of the following: **2×10**

5.	(a) Write down the features of soft computing. (b) Maximize $f(x) = 4 + 10x - x^2,\ 1 \le x \le 9$ using binary coded GA. Given that population size $N = 5$, initial population $x_1 = 10111$, $x_2 = 10101, x_3 = 11100, x_4 = 11101, x_5 = 10100$. Random numbers for selection: 0.19, 0.63, 0.97, 0.11, 0.70. Cross-over probability, $P_c = 0.8$ and random numbers for cross-over: 0.60, 0.85, 0.57, 0.37, 0.70. Mutation probability, $P_m = 0.04$ and random numbers for mutation: 0.21, 0.37, 0.02, 0.52, 0.07, 0.97, 0.14, 0.61, 0.17, 0.09, 0.03, 0.82, 0.08, 0.21, 0.37, 0.20, 0.25, 0.72, 0.24, 0.16, 0.47, 0.58, 0.49, 0.01, 0.18. (one iteration only)	2+8
6.	(a)Find the relational matrix of the concept "a young tall man", where "Young man"= $\frac{0}{115} + \frac{0.5}{120} + \frac{1}{125} + \frac{0.5}{130} + \frac{0}{135}$and "Tall man"= $\frac{0}{170} + \frac{0.5}{175} + \frac{1}{180} + \frac{1}{185} + \frac{1}{190}$, if possible with reason. (b) What do you mean by Fuzzy Inference System. Describe Mamdani's fuzzy inference method in short.	3+(2+5)
7.	(a) Mention the ranges of different GA parameters. (b) Using the perceptron learning rule, find the weights required to perform the following classifications {[(1, 1, 1), 0],[(-1, 1, 1), 0],[(-1, -1, 1), 1],[(-1, -1, -1), 1]}.	2+8
8.	(a) What is activation function? Mention two such activation function in neural network. (b) Generate the output of logical OR function by McCulloch-Pitts neuron model.	(2+2)+6

M.Sc.4th Semester examination, 2021
Department of Mathematics, Mugberia Gangadhar Mahavidyalaya
(Special Paper-OR: Nonlinear Optimization)
Paper MTM – 404B
FULL MARKS: 40 : : Time : 02 hours

*Answer any **Four** of the following questions* 4 X 10

1.	a) What is degree of difficulty in connection with geometric programming. b) Define: Nash equilibrium strategy and Nash equilibrium outcome. c) Define Pareto optimal solution in a multi-objective non-linear programming problem. d) State Kuhn-Tucker stationary point necessary optimality theorem. e) State Karlin's constraint qualification.	5x2
2.	(a) Write the relationsamong the solutions of MP, LMP, FJSP, KTSP. (b) State and prove Fritz-Johnsufficient optimality theorem.	4+6
3.	(a) Let θ be a numerical differentiable function on an open convex set $\Gamma \subset R^n$. θ is convex if and only if $\theta(x^2) - \theta(x^1) \leqq \nabla\theta(x^1)(x^2 - x^1)$ for each $x^1, x^2 \in \Gamma$. (b) Define the following terms: (i) The (primal) quadratic minimization problem (QMP). (ii) The quadratic dual (maximization) problem (QDP).	6+4
4.	(a) State and prove Motzkin's theorem of the alternative. (b) What is Nonvacuous matrix? State and prove Fritz John saddle point necessary optimality theorem.	5+(1+4)
5.	(a) How do you solve the following geometric programming problem? Find $X = \begin{Bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{Bmatrix}$ that minimizes the objective function $$f(x) = \sum_{j=1}^{n} U_j(x) = \sum_{j=1}^{N} \left(c_j \prod_{i=1}^{n} x_i^{a_{ij}} \right)$$ $c_j > 0,\ x_i > 0,\ a_{ij}$ are real numbers, $\forall\, i,\ j$(b) Derive the Kuhn-Tucker conditions for quadratic programming problem.	6+4
6.	(a) What are the basic differences between Polynomial & Posynomial?	2+3+5

(b) Explain Bi-matrix game.

(c) What is expected payoffs? Find the expected payoffs of two players

Strategy	t_1	t_2
s_1	(4,−4)	$(-1,-1)$
s_2	(0,1)	(1,0)

7. (a) What is stochastic programming problem? Give an example of stochastic programming problem? Write two important methods for solving stochastic programming problem? (1+1+1) +7

(b) Solve the following problem by Beale's method

$$\text{Maximize } z = 2x_1 + 3x_2 - x_1^2 - x_2^2$$
$$\text{subject to } x_1 + x_2 \leq 2$$
$$x_1,\ x_2 \geq 0$$

8. (a) State and prove Separation theorem. 5+5

(b) Solve the quadratic programming problem using Wolfe's modified simplex method

$$\text{Maximize } z = 2x_1 + 3x_2 - 2x_1^2$$
$$\text{subject to } x_1 + 4x_2 \leq 4,$$
$$x_1 + x_2 \leq 2,$$
$$x_1, x_2 \geq 0.$$

M.Sc.4th Semester examination, 2021

Department of Mathematics, Mugberia Gangadhar Mahavidyalaya

(Special Paper-OR: Lab. OR methods using MATLAB and LINGO)

Paper MTM – 495B

FULL MARKS: 25 : : Time : 02 hours

Group A

Answer one question 1x15=15

1.	Write a MATLAB program to solve the following LPP: $Max\ Z = 3x_1 + 5x_2$ $subject\ to$ $3x_1 + 2x_2 \leq 18$ $x_1 \leq 4$ $x_2 \leq 6$ $x_1, x_2 \geq 0$	15
2.	Write a program in MATLAB to solve the following Integer Programming Problem using Gomory's cutting plane method. Max $z = 3x_1 - 2x_2 + 5x_3$ Subject to, $5x_1 + 2x_2 + 7x_3 \leq 28$ $4x_1 + 5x_2 + 5x_3 \leq 30$ $x_1, x_2 \geq 0$and are integers.	15
3.	Write a MATLAB program to solve the following inventory problem: The demand for an item is 18000 units per year. The inventory carrying cost is Rs. 1.20 per unit per year and the cost of shortage is Rs. 5.00 per unit per year. The ordering cost is Rs.400.00 for each order. Assuming that the replenishment rate is instantaneous, determine the optimum order quantity, shortage quantity and cycle length.	15
4.	Write a program in MATLAB to solve the following Problem using Dynamic Programming technique. Max $z = y_1 y_2 y_3$ Subject to, $y_1 + y_2 + y_3 = 5$	15

	$y_1, y_2, y_3 \geq 0$	

Group B

Answer one question 1x10=10

5.	Write a LINGO program to solve the following queuing problem: A telephone exchange has two long distance operators. The telephone company finds that, during the peak hours, long distance call arrive in a Poisson fashion at an average rate of 15 per hour. The length of service on this call is approximately exponentially distributed with mean length 5 minutes. (i) What is the probability that a subscriber will have to wait for this long distance call during the peak hours of the day? (ii) If the subscriber waits and are serviced in turn, what is the expected waiting time.	10
6.	(a) Write the solution procedure and program in LINGO to solve the following Geometric Programming Problem. Minimize $f(x)=5x_1x_2^{-1} + 2x_1^{-1}x_2+5x_1+x_2^{-1}$ (b) Write the solution procedure and program in LINGO to find the Nash equilibrium strategy and Nash equilibrium outcome of the following bi-matrix game. **A** = $\begin{bmatrix} 8 & 0 \\ 30 & 2 \end{bmatrix}$ **B** = $\begin{bmatrix} 8 & 30 \\ 0 & 2 \end{bmatrix}$	5+5
7.	Write a LINGO program to solve the following QPP: $Maximize\ Z = 6x_1 + 3x_2 - x_1^2 + 4x_1x_2 - 4x_2^2$ $subject\ to$ $x_1 + x_2 \leq 3$ $4x_1 + x_2 \leq 9$	10
8.	Write the solution procedure and program in LINGO to solve the following LPP using Revised Simplex Method.	10

	Max $z = 3x_1+5x_2$ Subject to, $x_1 \leq 4$ $x_2 \leq 6$ $3x_1+2x_2 \leq 18$ $x_1, x_2 \geq 0$	